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복합 하중을 받는 단방향 판재들의 전단 응답에 대한 비선형성

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Nonlinearity of Shear Response of Unidirectional Laminates under Combined Loading

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초록: 본 논문에서는 보완된 변형 에너지 밀도 함수를 기반으로 하여 평면내 비선형 응력-변형률 곡선 모델을 제시한다. 다방향의 적측판에 전단방향과 축방향의 응력이 조합되어 작용할 때의 전단방향의 응답을 예측하였으며, 실험값과 비교하여 제시 모델의 정확성을 검증하였다. 이 과정에서 compliance 상수는 기존 축방향의 응력에 대한 측정값과 일치하도록 결정되었다. 마지막으로 제시된 모델을 이용한 E-glass/vinylester의 전단 응력-변형률 거동은 모든 축외 각도에 대하여 실험 곡선과 일치하며, boron/epoxy와 T800H/3633 적층판의 전단 거동은 15도를 제외한 30도, 45도, 60도의 축외 각도(off-axis angle)에 대하여 실험값과 일치한다. 이는 작은 적층 각도의 변화에도 종방향의 탄성계수는 다른 탄성계수에 비하여 매우 높기 때문이다.

Abstract: We present the prediction of nonlinear in-plane shear stress strain curves based on complementary strain energy density function. When the [θ°] unidirectional laminates are subjected to a combination of shear stresses and axial stresses, the predicted shear responses of laminates are compared with measured experimental data for verifying this accuracy of this model. In this process, the compliance constants are determined to match the measured test results in terms of the influence of on-axis axial stress. Finally, it is shown that the predicted shear stress strain behavior of E-glass/vinylester concurs with experimental curves for all off-axis angles, while shear behavior predictions of boron/epoxy and T800H/3633 laminates agree with experimental results for 30° 45° 60° off-axis angles laminates except 15° off-axis angle laminates. This is because the elastic longitudinal modulus is still significantly higher than the other modulus even for a small off axis angle.

Keywords: nonlinearity, in-plane shear, off-axis angles, GFRP, CFRP.

Introduction

As composite laminates are the widely used in practical engineering, the accurate analyses of mechanical properties are becoming important, so it is necessary to confirm the correct stress strain relationship in the failure analysis of composite laminates. There are actually numerous experimental results and theoretical studies on the stress strain relationship. In this work, mechanical responses of E-glass/vinylester, boron/epoxy and carbon/epoxy laminates are compared with theoretical predictions. The following three paragraphs are a review of three different laminates experimented by other researchers, and the remaining paragraphs in the section are different kinds of approaches to describe the nonlinear behavior.

The paragraph is intended to describe the mechanical behavior of E-glass/epoxy laminates. Aboudi[1] measured shear stress strain responses of unidirectional laminates, and also measured axial stress stain curves of angle-ply laminates (±15°, ±30°, ±35°, ±41°, ±45°). Haj-Ali[2] performed experiments to achieve the transverse compressive and shear responses of unidirectional laminates. Paepegem[3] investigated the in-plane shear response of [±45°] and off-axis [10°] stacking sequences lam-

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inates under tensile tests. Soden[4] listed experimental data of in-plane shear and transverse compressive behavior of E-glass/epoxy unidirectional laminates. The axial stress strain relationship of UD laminates seems to be linear while the hoop stress strain behavior shows the elastic – plastic performance, however, the axial stress strain response could be nonlinear when off-axis angles of UD laminates are nonzero. Comparing with the in plane shear stress train response, although it can be easily observed that the nonlinearity of hoop stress strain response exists at a high strain level, the nonlinearity of shear direction is so more significantly pronounced, no matter what the stacking sequence is UD or [±45°].

The mechanical properties of boron/epoxy laminates are investigated considering axial and in-plane shear directions. P. H. Petit[5] presented the stress strain responses in the longitudinal, transverse and in-plane shear directions, respectively. These laminates are classified as 0° unidirectional lamina, balanced and symmetric angle-ply laminates: [±20°], [±30°], [±60°], balanced and symmetric cross-ply laminates: [0°/90°] and quasi-isotropic laminates [0°/±45°]. Aboudi[1] presented axial stress versus axial strain and axial stress versus hoop strain curves of off-axis laminates (θ=15°, 30°, 45°, 60°) under uniaxial loading, and also presented the axial stress strain curves of angle-ply laminates (±20°, ±30°, ±45°, ±60°). The uniaxial stress strain relation of UD laminates is almost linear when the lamina is subjected to the axial loading, while the in-plane shear stress show significant nonlinearity. The axial stress versus axial strain and axial stress versus hoop strain curves are nonlinear for off-axis laminates, and the nonlinearity is more obvious as the off-axis angles are 15°, 30° and 45°. The uniaxial stress strain response of 60° off-axis laminates shows slightly nonlinear. When the stacking sequences are [±30°], [±45°], and [±60°], the nonlinear responses of uniaxial stress strain are easily to be observed, while response of [±20°] laminates is nearly to be linear.

As for studies of mechanical behavior of carbon/epoxy laminates composed of different classes of fiber (AS4, T300, T700), Soden[4] presented experimental data and figures for the properties of the two kinds of AS4/3501 and T300/BSL914C laminates. The stress strain curves are slightly nonlinear at a high strain levels under the conditions of longitudinal tensile loading and transverse compressive loading, respectively. And in-plane shear stress strain curves show very significantly nonlinear properties. Ogihara[6] investigated the mechanical behavior of T700S/2500 unidirectional and angle-ply laminates under uniaxial tensile loading. For unidirectional laminates except [0°] laminates, all axial stress strain curves show obvious nonlinearity, and the stress strain curves for angle-ply laminates except [±15°] also show nonlinearity, in particular for [±45°] laminates.

In order to get an accurate stress strain relationship of composite laminates, both macromechanical models and micromechanical models are proposed to predict the linear slope at a low strain level and nonlinear curve at a high strain level.

Ramberg-Osgood method is widely used for describing elastic-plastic stress strain relationship, moreover, the model can imply material plastic response at a low strain level. In order to derive this function, a yield offset value is introduced so as to satisfy the elastic stress strain relation at a reference stress σ_0. There are indeed numerous studies and tests performed on this formula, nevertheless, many expressions of the formula are provided by that strain are derived from stress with initial shear modulus and compliance constants consisted of three adaptations of material parameters,[2,7] while the Bogetti[8] and Richard[9] proposed another Ramberg-Osgood model that the stress is expressed by the strain with initial shear modulus, asymptotic stress, and a shape parameter. Comparing with the previous formula, the advantage of this formula is that the prediction from the formula can match the experimental data well when the stress almost rarely change with increasing strain. The reason why the differences exist between the two kinds of expressions is that the formula expressed by stress cannot be derived for the interval in which stress almost keeps the same with the increasing strain, based on mathematical explanation.

The above model is based on physical phenomenon, which is derived from the experimental curves. The following discussions are related to the mechanical behavior of laminates considering that failure of laminates is controlled by transverse matrix crack or inter-fiber failure.

Sun[10] and Majid[11] described the shear stress strain relation by using the determination of effective shear modulus. In the failure analysis of laminates, they took first initial failure and subsequent progression of matrix cracking into account, hence, the stiffness degradation is needed to be verified. In order to achieve the effective stiffness, initial modulus and crack density as well as a curve fitting parameter are necessary.

Puck[12] thought that hoop stress has an influence on the risk of fracture, hence, the stress exposure f_E is recommended. In his research, a computational method is proposed by means of Puck's failure criteria with the consideration of an influence of the normal stress on the shear strain. The author assumed that the nonlinear behavior occurred until the stress exposure f_E

exceeded a threshold value f_{Ethr}. According to the comparisons of stress strain relationship between measured experiments and calculations, the author recommend these values of the three numerical adaptation of parameters (f_{Ethr}, n, C), and there is still two parameter need to be verified by using failure criteria. In the model, the stress exposure f_E is decisive parameter.

There is also a model based on the definition of complementary strain energy density. Researchers discussed the applications of the model by means of comparing the theoretical predictions with experiments. Hahn and Tsai[13] discussed the nonlinear shear stress strain relationship of material boron/epoxy without coupling uniaxial stress and in plane shear stress under on-axis coordinates and off-axis coordinates. It is observed that longitudinal tensile modulus changes with tensile stress in study of Ishikawa *et al.*.[14] The fitting curves match the test for carbon composites when the laminates are under longitudinal tensile loading. Luo[15] employed the complementary energy method for flexible composites composed of continuous fibers in an elastomeric matrix. Axial stress strain responses of materials tirecord/rubber, Kevlar-49/silicone are compared with the experiments for off-axis specimen. The correlation between predicted response and experiment is good. Kroupa[16] performed tensile tests of carbon/epoxy with various fiber orientations and identified the material parameters.

The above are macromechanical modes that identified parameters by fitting the experimental data. Certainly, there are also micromechanical approaches analysed by a generalized cells model. The cell model is based on a rectangular cell constituted four square sub-cells (fiber and matrix) along with the cell geometry. The strains are the same in the sub-cells and the sum stress of the sub-cells equals the stress of the cell, as a result, it is proposed that mechanical behavior of laminates is denoted by the overall average values. The procedure of derivation and verification of this model are reported by Haj-Ali,[2] Aboudi[1] and Paley.[17]

Theory

Estimation of Constitutive Stress Strain Relation. The constitutive rule is derived from complementary strain energy density function. Since the forces should keep a state of equilibrium when laminates are under static conditions, the virtual work of internal forces equals the virtual work of external work. Hence, it leads that the derivative of strain energy can be expressed by the derivative of displacement component and the corresponding force component. In process of the achieving strain energy by means of integrating the derivative of strain energy, the function called complementary energy is introduced. For the sake of simplification, the case of small deformation is taken into account. The strain energy density function W and the complementary strain energy density W^* are proposed.[18] In the theory of elasticity, the stress-strain relation can be written as in the following equation,[13,18]

$$\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} \tag{1}$$

Since strain components are not independent, which can be regarded as functions of the stress components, then the complementary strain energy density W^* is given by integrating both sides of the eq. (1).

$$W^* = \sigma_{ij}\varepsilon_{ij} - W \tag{2}$$

Therefore, strains can be calculated by

$$\varepsilon_{ij} = \frac{\partial W^*}{\partial \sigma_{ij}} \tag{3}$$

where the subscripts (i, j) denote the plane on which the stress act and the direction of the component to the plane, respectively. In the study, both of the two indices (i, j) are assumed to range over 1, 2 for the two-dimensional plane stress condition. Material coordinate axis x_1 is parallel to the fiber direction, and x_2 is perpendicular to the fiber direction. And assuming all fibers are aligned parallel to the fiber direction in the same lamina, this means the effects of fiber misalignment[19] and fiber waviness[20] on the performances of laminates are not considered in the analysis. The complementary energy density W^* with planes stresses $\{\sigma_1, \sigma_2, \sigma_6^2\}$ is expressed by the form of polynomial expansion in terms of high order terms,

$$\begin{aligned} W^* = {} & \frac{1}{2}S_{11}\sigma_1^2 + \frac{1}{2}S_{22}\sigma_2^2 + \frac{1}{2}S_{66}\sigma_6^2 + S_{12}\sigma_1\sigma_2 + \frac{1}{3}S_{111}\sigma_1^3 + \frac{1}{3}S_{222}\sigma_2^3 + \\ & S_{112}\sigma_1^2\sigma_2 + S_{122}\sigma_1\sigma_2^2 + S_{166}\sigma_1\sigma_6^2 + S_{266}\sigma_2\sigma_6^2 + \frac{1}{4}S_{1111}\sigma_1^4 + \frac{1}{4}S_{2222}\sigma_2^4 + \\ & \frac{1}{4}S_{6666}\sigma_6^4 + S_{1112}\sigma_1^3\sigma_2 + S_{1122}\sigma_1^2\sigma_2^2 + S_{1222}\sigma_1\sigma_2^3 + \mathrm{S}_{1166}\sigma_1^2\sigma_6^2 + \\ & S_{2266}\sigma_2^2\sigma_6^2 + S_{1266}\sigma_1\sigma_2\sigma_6^2 \end{aligned} \tag{4}$$

If the coupling effects between axial stresses and shear stresses are negligible, the corresponding third-order and the fourth-order constants should be zero. And if further simplification are made, all stress strain relationship should be linear no matter what the load is applied. As a result, the complementary energy function is written in the following.

$$W^* = \frac{1}{2}S_{11}\sigma_1^2 + \frac{1}{2}S_{22}\sigma_2^2 + \frac{1}{2}S_{66}\sigma_6^2 + S_{12}\sigma_1\sigma_2 \quad (5)$$

The eq. (5) is commonly used. It is known that terms S_{ij} are elastic properties, which are determined from the initial stress strain slopes, for example, S_{11} is derived from the expression, $S_{11} = d\sigma/d\varepsilon = 1/E_1$, so the other S_{ij} coefficients are the reciprocal of the initial elastic modulus of composite laminates except the term S_{12}. When nonlinearity of material laminates are considered, the third-order and the fourth-order compliance coefficients are determined by fitting the predicting curves with experimental results. And S_{166}, S_{266}, S_{1166}, S_{2266}, S_{1266} are items that describing the non-linear coupling effects representing the interactions between axial and shear deformations. The item S_{6666} is a decoupling parameter, which is only related to pure shear stress.

It is summarized form the above that eq. (4) is an unabridged expression for the complementary strain energy density. Assuming that the shear stress strain curves keep the same even when the laminates are subjected to the opposite sign and the same magnitude of loads, it leads that parameters S_{166} and S_{266} should be zero. Then the constitutive relationship for in-plane shear stress strain is built by substituting eq. (4) into eq. (3), shown in eq. (6).

$$\varepsilon_6 = S_{66}\sigma_6 + S_{6666}\sigma_6^3 + 2S_{1166}\sigma_1^2\sigma_6 + 2S_{2266}\sigma_2^2\sigma_6 + 2S_{1266}\sigma_1\sigma_2\sigma_6 \quad (6)$$

From the equation, it is easily seen that the shear stress strain relationship is nonlinear, and this phenomenon can be clearly observed in the experimental data. In many studies, researchers ignore the influence of axial stress on shear property, only pure shear stress are considered.

The longitudinal strength of laminates is controlled predominantly by the fiber strength, while the transverse strength and in-plane shear strength are mainly influenced by the matrix and the interfacial bond strength. Nonlinear behavior of pure epoxy is investigated by Benzerga[21] and Fiedler.[22] Thermoset epoxy resin LY556 and resin 113 are tested, respectively. It is obvious to observe that the epoxy shows elastic properties at a low strain level and plastic properties at a high strain level when the epoxy are applied to uniaxial tension, uniaxial compression and torsion. It can be assumed that the nonlinear material properties are attributed to the matrix. As a consequence, the transverse stress strain curves and in-plane shear stress-strains are more significantly nonlinear comparing with longitudinal stress strain curves. Figure 1 shows the in-plane shear stress-strain responses under combined loading of transverse and shear stress.[12,23] Seen from Figure 1(a), comparing with shear stress response of laminates under only pure shear stress, shear stress strain curve is up under combining moderate transverse compressive stress and shear

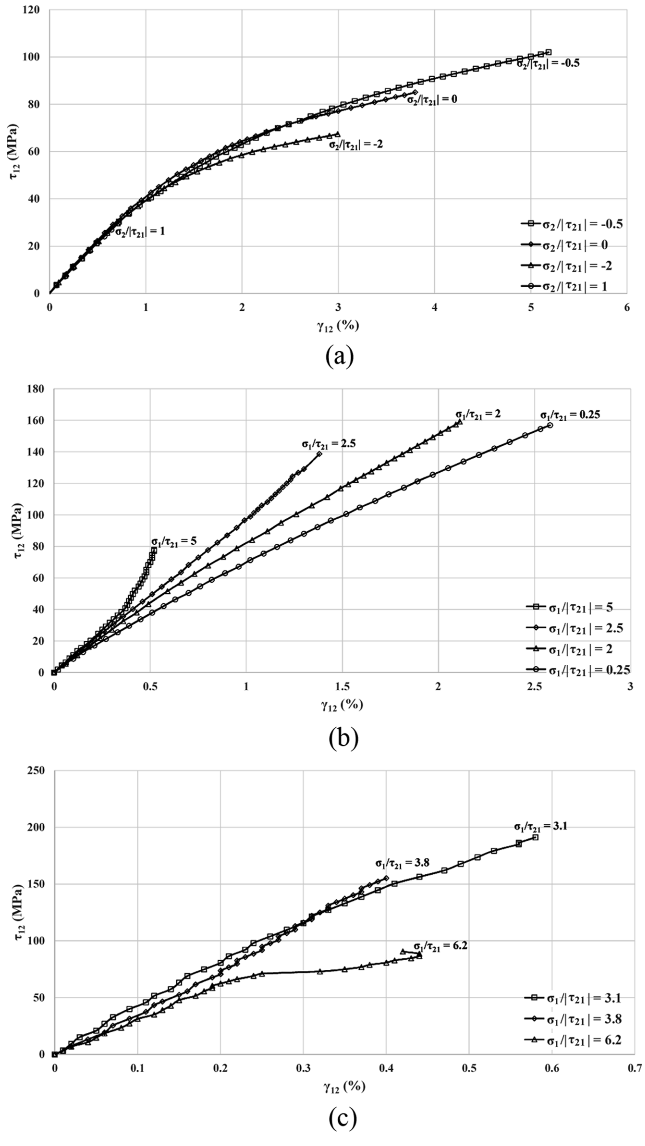

Figure 1. In-plane shear stress-strain (τ_{12}, γ_{12}) curves of S-glass/epoxy and carbon/epoxy laminates under different combined loads. (a) measured shear curves for carbon/epoxy (T300/LY556) laminates under different axial stress σ_2 to shear stress τ_{12} ratios; (b) shear stress strain responses of [90°/-30°/+30°/90°/90°/+30°] S-glass/epoxy laminates under different axial stress σ_1 to shear stress τ_{12} ratios; (c) shear stress strain responses of [+30°/-30°] carbon/epoxy laminates under different axial stress σ_1 to shear stress τ_{12} ratios.

stress, while shear stress strain curve is down under combing the transverse tensile stress or high transverse compressive and shear stress. Seen from Figure 1 (b) and 1(c), it is apparent that shear responses are different under various axial stress to shear stress ratios. The shear stiffness of S-glass laminates increases with the axial stress to shear stress ratio. Furthermore, the initial shear modulus of carbon/epoxy laminates decreases as the stress ratio increases.

The aim of the present study is to describe the nonlinear shear stress strain relationship in terms of the influence of longitudinal stress and transverse stress. This could provide more accurate failure prediction of laminates, especially for matrix failure and inter fiber failure.

Estimation of the Shear Stress and Shear Strain. It is obvious that the contraction in transverse direction occurs while elongating in the longitudinal direction under uniaxial loading conditions. But for sake of simplifying experimental procedures or the restriction in experimental apparatus, generally, the strain gauge attached to the surface of specimen is used to measure the corresponding stress strain curves caused by the external loads. So the axial stress versus axial strain curves are the very common while axial stress versus hoop strain curves are less common, but the hoop strains always exist as long as there are longitudinal strains. As a result, in order to accurately and comprehensively understand responses of fiber reinforced plastic plates, it should be convenient to make use of the off-axis elastic properties, namely off-axis Poisson's ratio verified by the eq. (7) by making use of transformed compliance matrix based on classic laminate theory.[24] The off-axis elastic modulus is obtained by the same way, expressed in eq. (8). Where m equals cosθ; n equals sinθ; E_1, E_2, ν_{12}, and G_{12} are in-plane elastic properties.

Table 1. Elastic Properties of Fiber Reinforced Plastic Laminates

Ply angles	E_{11} (GPa)	E_{22} (GPa)	ν_{12}	G_{12} (GPa)
E-glass/vinylester	19.31	12.61	0.3	4.50
Boron/epoxy	206.85	22.06	0.23	7.24
T800H/3633	145.8	9.26	0.358	5.57

$$\overline{\nu_{12}} = \frac{(n^4+m^4)\nu_{12} - m^2n^2(1+E_1/E_2-E_1/G_{12})}{m^4+m^2n^2(-2\nu_{12}+E_1/G_{12})+n^4E_1/E_2} \quad (7)$$

$$\overline{E_1} = \frac{E_1}{m^4+m^2n^2(-2\nu_{12}+E_1/G_{12})+n^4E_1/E_2} \quad (8)$$

In this study, three different [θ°] unidirectional laminates are selected to predict the nonlinear shear responses considering the influence of axial stresses on the shear stress strain curves. The elastic properties of the three laminates are listed in Table 1. And according to eq. (7) mentioned above, the off-axis Poisson's ratios of E-glass/vinylester and T800H/3633 laminates are calculated for different ply angles. As for the off-axis Poisson's ratios of boron/epoxy, they are obtained from the measured experimental data. All the values are summarized in Table 2. And the off-axis Poisson's ratio are elastic for all laminates. The off-axis elastic modulus are presented in Tables 3.

For purpose of clear and straightforward display of mechanical properties, the graphical measures of uniaxial stress strain curves are showed in Figure 2. The axial stress versus axial strains of [θ°] off-axis laminates are presented in I quadrant, while the axial stress versus the hoop strain obtained data based on the Poisson's ratios are showed in II quadrant. It is

Table 2. The Off-axis Poisson's Ratios of Different Off-axis Angles Laminates

Ply angles	0°	15°	30°	45°	60°	90°
E-glass/vinylester	0.300	0.341	0.397	0.401	0.344	0.211
Boron/epoxy	0.23	0.5	0.58	0.49	0.39	0.025
T800H/3633	0.358	0.447	0.427	0.343	0.216	0.023

Table 3. Elastic Modulus (E_{11}(GPa)) of Different Off-axis Angles Laminates

Ply angles	0°	15°	30°	45°	60°	90°
E-glass/vinylester	19.31	17.47	14.39	12.51	12.04	12.61
Boron/epoxy	206.85	77.49	32.22	21.50	19.50	22.06
T800H/3633	145.80	57.57	23.07	13.82	10.65	9.26

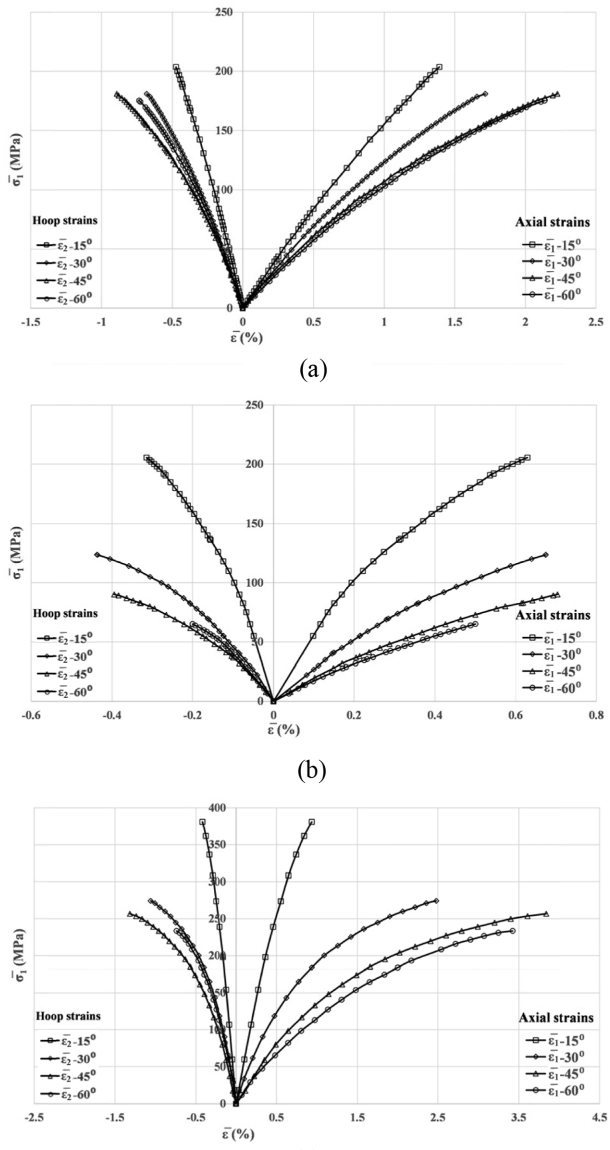

Figure 2. Uniaxial stress strain response of [θ°] off-axis laminates. (a) for E-glass/vinylester laminates; (b) for boron/epoxy laminates; (c) for T800H/3633 laminates.

evident that behaviors of uniaxial stress strain curves are elastic-plastic, moreover, this phenomenon is more obvious for boron/epoxy and T800H/3633 laminates. The axial stress strain curves are more significantly shifted downward when the off-axis angles increases from 15° to 30°, comparing with the conditions when the off-axis angles increases from 30° to other degrees. And also this initial slope of the stress strain curve gives a measure of the elastic modulus, which should be consistent with the corresponding data filled in the Table 3.

The above part is mainly to propose the approach for obtaining off-axis elastic properties, that is, the hoop strains can be obtained in the laminate coordinate system. And it is known that the relationship between external forces and displacements is analyzed in global coordinate system, however, the material properties are specified in local coordinate system. Hence, although the unidirectional laminates are subjected to uniaxial loading, the [θ°] unidirectional laminates may be under a set stresses $\{\sigma_1, \sigma_2, \sigma_6\}$ in material coordinate system, as depicted in Figure 3. Where $\overline{x_1}$, $\overline{x_2}$ axes and x_1, x_2 axes are global coordinate and local coordinate, respectively; the axis x_1 is aligned with the fiber direction; θ° is ply angle. In order to get the on-axis stresses and strains, the two transformation matrix functions are needed, as displayed in the following eqs. (9), (10).

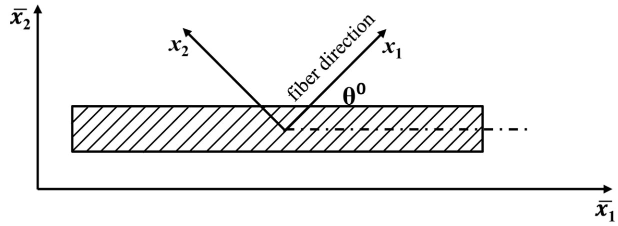

Figure 3. The schematic of fiber reinforced plastic laminates with off-axis angle θ°.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{Bmatrix} \overline{\sigma_1} \\ \overline{\sigma_2} \\ \overline{\sigma_6} \end{Bmatrix} \quad (9)$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{Bmatrix} \overline{\varepsilon_1} \\ \overline{\varepsilon_2} \\ \overline{\varepsilon_6} \end{Bmatrix} \quad (10)$$

Results and Discussion

According to the axial and hoop strains achieved from the previous part, shear strains can be calculated in the local coordinate system by multiplying with the transformation matrix, that is, the shear strain can be given by the equation $\varepsilon_6 = -2\sin\theta\cos\theta \cdot \overline{\varepsilon_1} + 2\sin\theta\cos\theta \cdot \overline{\varepsilon_2}$. The analytical data for shear stress strain is derived from eq. (7). The compliance constants are determined by fitting the shear stress strain curves to match the experimental behavior. Parameters S_{66} (Pa^{-1}) and S_{6666} (Pa^{-3}) are obtained from the responses of unidirectional laminates under pure shear loading, and the other three remaining parameters are obtained for laminates under off-axis loading. All material parameters are summarized in Table 4 for the three different laminates. Figures 4-6 are graphical comparisons between the experimental data and theoretical results of the three laminates.

It can be concluded from the figures that all measured experimental curves exhibit significantly nonlinear in-plane behavior, in particular for boron/epoxy and T800H/3633 laminates with higher elastic longitudinal modulus. The agreement between analysis and experimental data for E-glass/vinylester is better than for the other two laminates. Among all cases under different off-axis loadings of the same material laminates, the comparisons from laminates with stacking sequence 30°, 45°, 60° off-axis angles show better agreement than results from 15° off-axis angle laminates. When E-glass/vinylester

Table 4. Measured Mechanical Properties of the Three Laminates

Constants	S_{66}(Pa^{-1})	S_{6666}(Pa^{-3})	S_{1166}(Pa^{-3})	S_{2266}(Pa^{-3})	S_{1266}(Pa^{-3})
E-glass/vinylester	1.90×10^{-10}	2.15×10^{-26}	-6.58×10^{-28}	1.60×10^{-27}	-2.73×10^{-27}
Boron/epoxy	1.41×10^{-10}	1.84×10^{-26}	-1.15×10^{-27}	1.71×10^{-26}	-3.71×10^{-27}
T800H/3633	1.70×10^{-10}	4.96×10^{-26}	-7.94×10^{-28}	1.35×10^{-27}	-1.93×10^{-26}

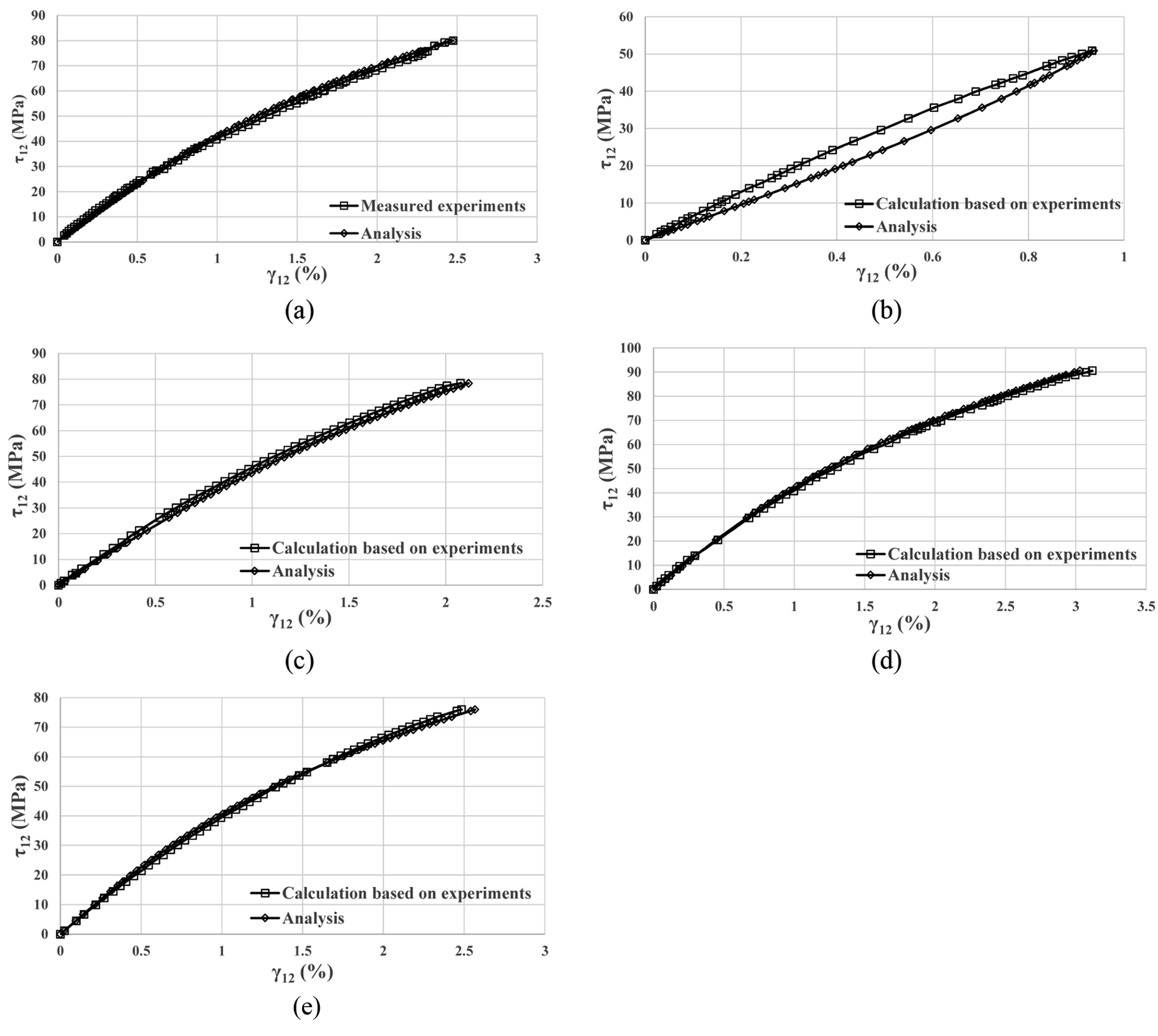

Figure 4. Comparisons of shear stress strain responses of E-glass/vinylester between data based on experiments and data based on theory for different off-axis loadings ((a) 0°; (b) 15°; (c) 30°; (d) 45°; (e) 60°).

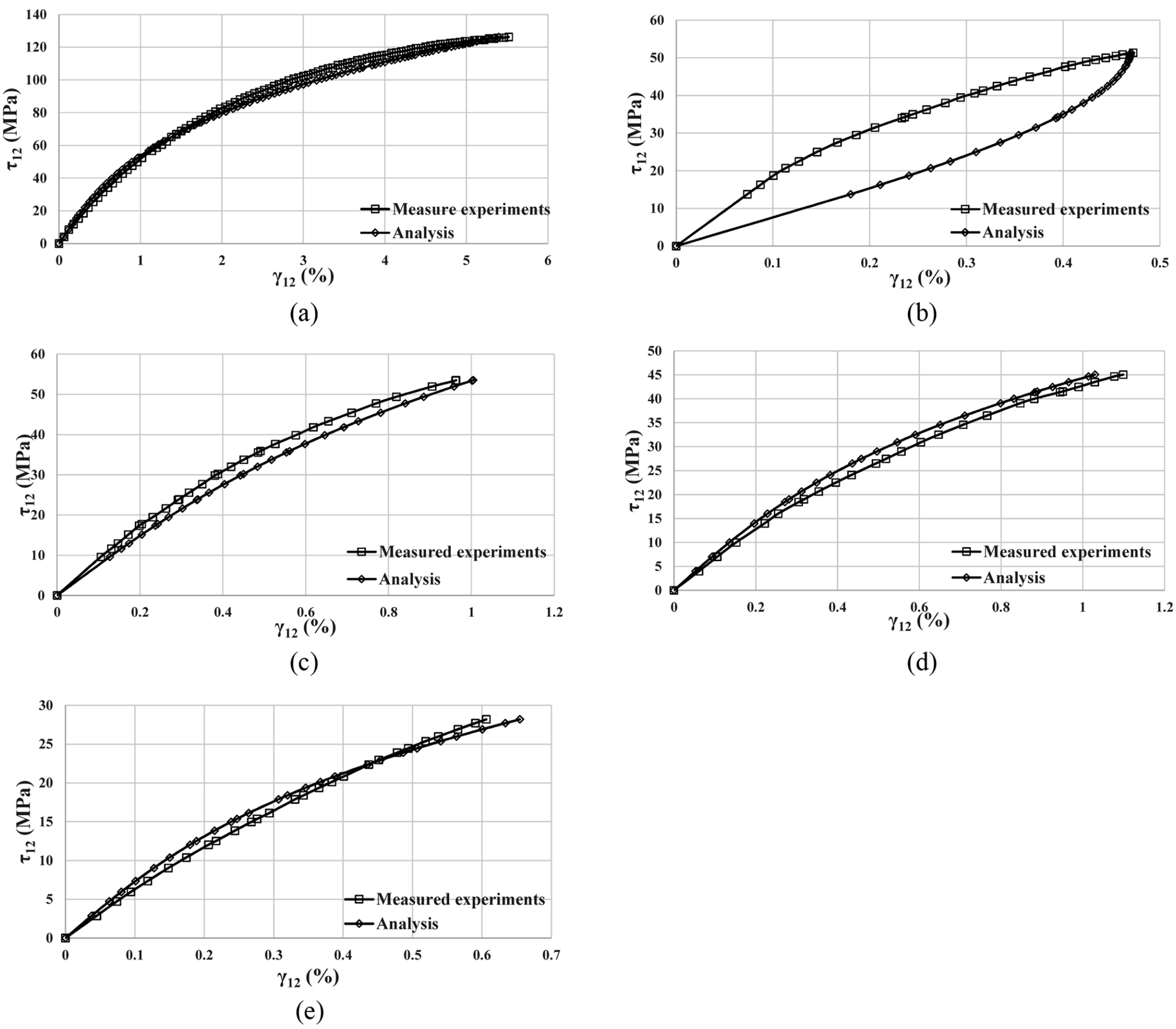

Figure 5. Comparisons of shear stress strain responses of boron/epoxy between measured experiments and data based on theory for different off-axis loadings ((a) 0º; (b) 15º; (c) 30º; (d) 45º; (e) 60º).

laminates are under combined loadings (σ_1, σ_2, τ,) at 30º, 45º, 60º off-axis angles, the predicted curves match measured experimental values very well, while in the case of boron/epoxy and T800H/3633 laminates, the analysis does not fit experiments as well as results from E-glass/vinylester laminates, but the predicted curves show the same nonlinear response as experiments, in addition the difference is in the tolerance range.

Seen from Figure 4(b), Figure 5(b) and Figure 6(b), the theoretical predictions are lower than the experimental data, and there is quite a difference for boron/epoxy and T800H/3633 laminates at 15º off-axis angle. The initial slope of shear stress strain curve of 15º off-axis laminates resulting from theoretical analysis is mainly determined by parameter S_{66}, which is inverse of elastic shear modulus. However, the nonlinear shear responses of [θº] laminates in local coordinate are derived from experimental results of uniaxial stress responses of laminates in the global coordinate, it is necessary to illustrate Figure 4(b), Figure 5(b) and Figure 6(b) by means of Figure 2 depicted by experiments. According to stress strain curves showed in Figure 2, the difference of stress strain slopes between 15º off-axis laminates and other angles off-axis laminates is much higher for boron/epoxy and T800H/3633 laminates than for E-glass/vinylester when the three different laminates are subjected to uniaxial loading. There is no doubt that superior performances of composite laminates are due to fiber properties, hence, elastic longitudinal modulus of boron/epoxy and T800H/366 are higher than modulus of E-glass/vinylester, moreover, the ratios of longitudinal to other elastic modulus of the two materials are also higher than that from E-glass/vinylester. The com-

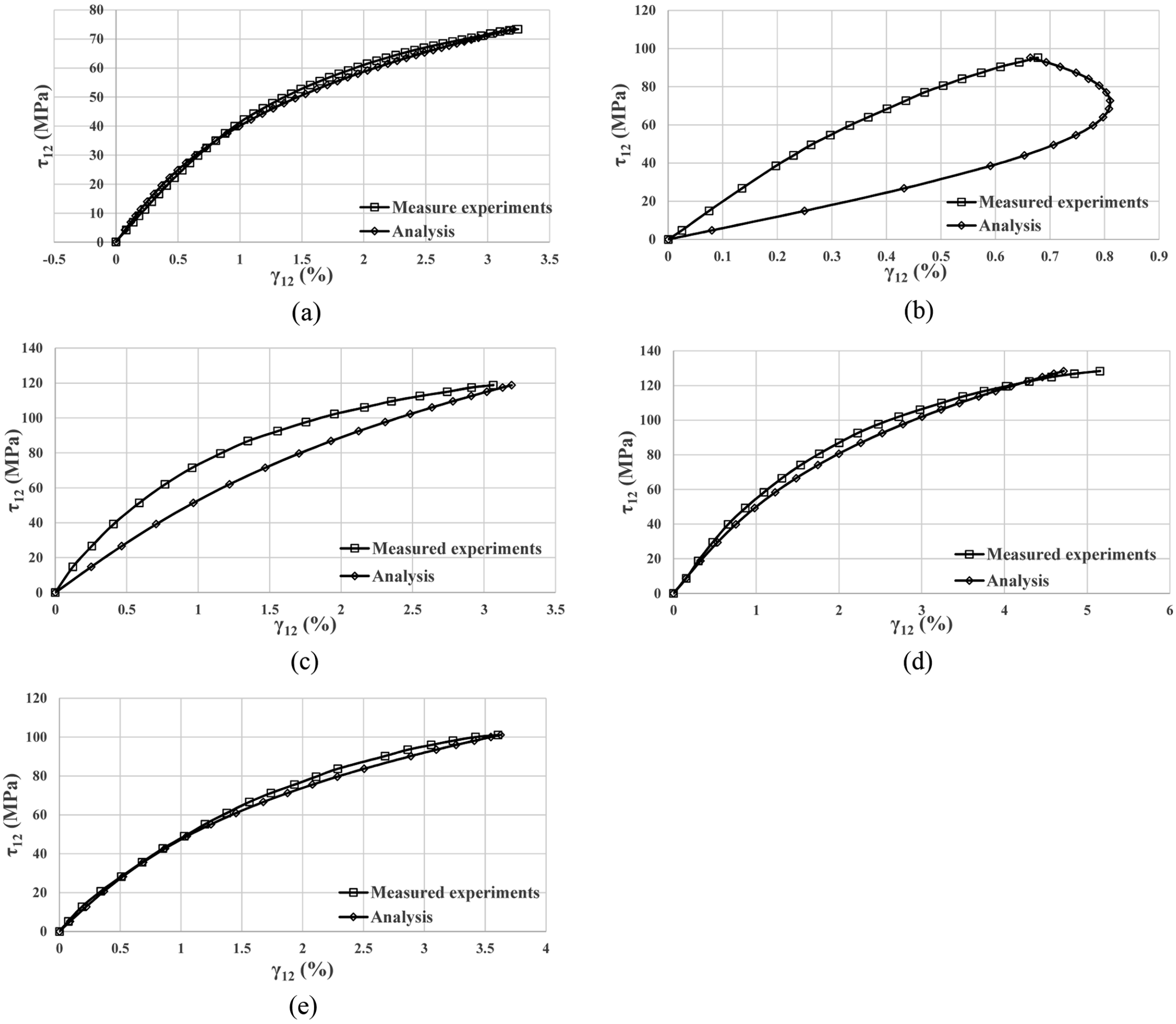

Figure 6. Comparisons of shear stress strain responses of T800H/3633 between measured experiments and data based on theory for different off-axis loadings ((a) 0º; (b) 15º; (c) 30º; (d) 45º; (e) 60º).

posite laminates are regarded as transversely isotropic materials. Although laminates with smaller off-axis angles still carry more loading, as the off-axis angles increase, the ability to carry longitudinal loading becomes weak and the ability to carry transverse loading becomes enhanced. Therefore, the longitudinal modulus of 15º off-axis laminates consisted of boron/epoxy and T800H/366 is quite a higher value than modulus of 30º, 45º, 60º off-axis laminates, which is different from E-glass/vinylester laminates. This is also can be observed in Figure 2 and Table 3.

Hence, longitudinal properties has a more influence on axial stress strain curves of 15º off-axis laminates. The longitudinal performances of the three materials are advantageous over transverse performances, the characteristic is so apparent for boron/epoxy and T800H/3633 laminates. This is the reason why analysis for boron/epoxy and T800H/3633 laminates do not fit experiment of the shear stress strain curve at 15º off-axis angle. Moreover, it is known that in plane shear response appears to be influenced less by axial stress than by hoop stress, which is also reflected in absolute values of material constants S_{1166} and S_{2266} used in polynomial expansion of shear strain.

Conclusions

A nonlinear constitutive relation between in plane shear stress and strain is proposed based on the complementary strain energy density function. The influences of axial stress and hoop stress are taken into consideration in shear response by means of material parameters. Hence, it is convenient to fit the experiments when laminates are under combined loads. When the failure mode is matrix failure or inter-fiber failure, this would provide an accurate failure analysis results of laminates. The three different [θº] unidirectional laminates are used to verify the computational model. All agreements between theoretical results and measured experiments are satisfactory, except the case when 15º off-axis boron/epoxy and T800H/3633 laminates are under uniaxial loading. The initial slopes of predicted shear stress strain curves are mainly determined by the elastic shear modulus. While the experimental data are derived from [θº] laminates are under uniaxial stress, that is, the initial slope is influenced a lot by elastic longitudinal modulus. The magnitudes of elastic longitudinal modulus of boron/epoxy and T800H/3633 laminates are so large that the longitudinal modulus is still significantly higher than the other elastic modulus when laminates are stacked by smaller angles. Hence there is a difference between predictions and experiments for 15º off-axis boron/epoxy and T800H/3633 laminate while fitting curves generated by material constants match the corresponding experiments for other off-axis angles at the same time. In the end, the computational model can predict nonlinear shear stress in terms of the influence of axial stress and hoop stress when the materials are glass fiber reinforced plastic, and the model can apply for carbon fiber reinforced plastic when the off-axis angle is not small.

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