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Theoretical Analysis of Bending Phenomenon at the Die Exit in Bicomponent Polymer Melt Flow —The Effect of the Melt Elasticity Difference—

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Abstract: The bending phenomenon at the die exit is theoretically analyzed in the case of bicomponent polymer melt flow. The bending angle was derived from the melt elasticity difference of the two phases and determined as a function of the phase portion and the ratio of extrudate swelling. It was found that the exit bending angle increased with the ratio of extrudate swelling and that the angle had a maximum when it was plotted against the phase portion. The maximum value increased with the ratio of extrudate swelling and shifted to the high value of higher elastic phase portion as the ratio of extrudate swelling increased. The interface shifting phenomenon at the die exit is also discussed briefly by geometrical analysis.

1. Introduction

In recent years, coextrusion has emerged into the polymer processing industry as means of economically producing composite films and conjugate fibers[1]. An industrial importance of coextrusion is to form a multilayer film having unique optical and mechanical properties. To the fiber industry, the stratified flow of two polymer melts through a die can result in the spinning of bicomponent fibers which are important for their self-crimp characteristics[2]. A technique of suede[3,4] from multicomponent fiber spinning is now achieved routinely on industrial scale.

The success of coextrusion process depends on the die design. There are a number of patents[5] that describe die designs and methods of manufacturing of bicomponent fibers. Several researchers have reported fundamental studies and experimental results on the bicomponent polymer melt flow.

The interface movement of bicomponent polymer melt flow was studied by Russell[6], and White[7]. Russell derived the interface velocity and stress by using force balance method and White carried out an experimental and theoretical study on the interface velocity in bicomponent polymer melt flow. Han published two papers dealing with coextrusion in 1973[8,9]. He performed experiments with the thin slit die. He discussed the effect of viscosity and elasticity of flow behavior and discussed the shape of the interface between two phase in flow.

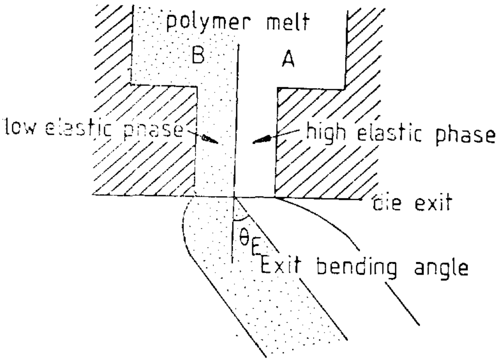

Fig. 1. Bicomponent polymer melt flow.

The bending phenomenon of bicomponent polymer melt at the die exit is one of recent subjects. It is generally recognized that the exit bending angle caused by elastic effect takes place towards the higher elastic phase, as shown in Fig.1. In 1979, Southern[10,11] reported a research of the exit bending phenomenon considering three factors; difference in melt viscosity, difference in melt elasticity, and difference in surface tension of two phases. He explained the exit bending phenomenon on the basis of experimental results, but did not attempt to analyze theoretically.

In our previous paper[12], the exit bending angle was theoretically derived from the viscosity difference of two phases only, and was determined as a function of melt viscosity ratios, phase portions and die dimensions. In this paper, the effect of melt elasticity difference (die swell ratios) on the exit bending angle is theoretically analyzed assuming that the differences in viscosity and surface tension are neglected. The interface shifting phenomenon is also discussed briefly by geometrical analysis

2. Theoretical Treatment and Discussion

In order to derive conveniently the theoretical exit bending angle caused by elastic effect, the followings are assumed:

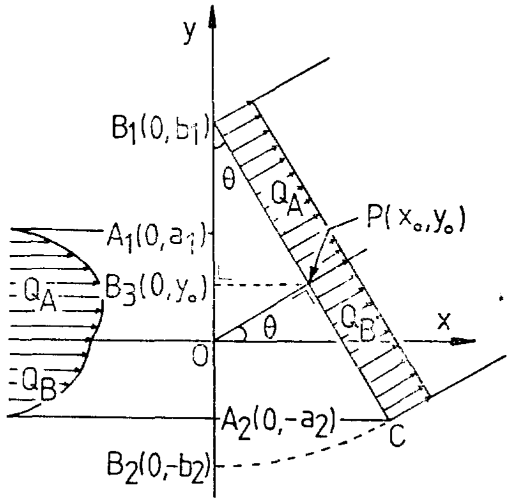

Fig. 2. Geometry of positive bending angle.

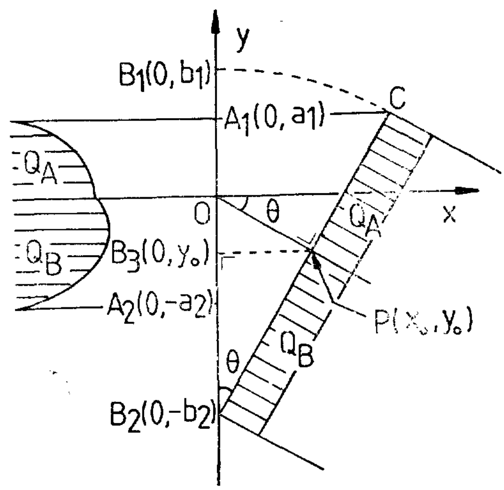

Fig. 3. Geometry of negative bending angle.

(1) The effects of viscosity and surface tension are neglected.

(2) The effect of relaxation time on swelling is neglected.

(3) The two phases are under equal pressure drop.

(4) The flow after die exit is considered as a plug flow.

Two variables are defined for extrudate swelling (Figs.2 and 3) : die swell ratio, $(d_j/$

$D_0)_A = b_1/a_1$ and $(d_j/D_0)_B = b_2/a_2$; ratio of extrudate swelling, $s=(d_j/D_0)_A/(d_j/D_0)_B$.

2.1 Derivation of the Exit Bending Angle θ_E

In Fig.2, letting a interface $\overline{OP}$ be perpendicular to the base line $\overline{B_1C}$ of the plug flow, the exit bending angle becomes the angle θ between the interface $\overline{OP}$ and the x axis. If we set $\tan\theta = m$, then $m = y_0/x_0$. At the point P (x_0, y_0) on the base line $\overline{B_1P}$ of the plug flow,

$$y_0 - b_1 = -x_0/m \tag{1}$$

Let the line $\overline{B_3P}$ be perpendicular to the die exit line (y axis). Applying the Pythagorean theorem on $\triangle B_1B_3P$ results in

$$(b_1 - y_0)^2 + (x_0)^2 = (\overline{B_1P})^2 \tag{2}$$

$\overline{B_1P}$ is obtained from the condition that the volumetric flow rate does not change as polymer melts flow.

$$\overline{B_1P} = (b_1 + b_2)\cdot q_A \tag{3}$$

Substitution of Eqs.(1) and (3) into Eq. (2) gives

$$(x_0/m)^2 + (x_0)^2 = (b_1+b_2)^2\cdot(q_A)^2 \tag{4}$$

From Eqs.(1) and (4), one can obtain the position of point $P(x_0, y_0)$

$$x_0 = \frac{(b_1+b_2)\cdot q_A}{\sqrt{1+(1/m)^2}} \tag{5a}$$

$$y_0 = \frac{(b_1+b_2)\cdot q_A}{m\sqrt{1+(1/m)^2}} + b_1 \tag{5b}$$

Using Eqs.(5a) and (5b), one can obtain the following expression(Eq.6) for the slope of swollen flow.

$$\frac{y_0}{x_0} = m = -\frac{1}{m} + \frac{b_A}{q_A}\sqrt{1+\frac{1}{m^2}} \tag{6}$$

Rearrangement of Eq.(6) gives

$$m + \frac{1}{m} = \frac{b_A}{q_A}\sqrt{1+\frac{1}{m^2}} \tag{7}$$

In the case of $m \geqq 0$, the Eq.(7) becomes

$$m^2+1 = \frac{b_A}{q_A}\sqrt{m^2+1} \tag{8}$$

Putting $m^2+1=t$, then the Eq.(8) becomes

$$t = \frac{b_A}{q_A}\sqrt{t} \tag{9}$$

Solving the Eq.(9), one can obtain the slope m of interface of the flow which has left the die exit.

$$t = (b_A/q_A)^2 \tag{10}$$

$$m = \sqrt{(b_A/q_A)^2 - 1} \tag{11}$$

Hence, the bending angle θ is determined from Eq.(11).

$$\theta = \tan^{-1}\sqrt{(b_A/q_A)^2-1} \tag{12}$$

In Eq.(12), one can easily see that $(b_A/q_A) \geqq 1$. If $0<(b_A/q_A)\leqq 1$, it is equivalent to the condition $(b_B/q_B)\geqq 1$ (Fig.3). Similarly, in the case of $(b_B/q_B)\geqq 1$, one can obtain the bending angle θ by using of Pythagorean theorem of $\triangle B_2B_3P$ and the plug flow condition.

$$\theta = -\tan^{-1}\sqrt{(b_B/q_B)^2-1} \tag{13}$$

Using the characteristics of unit step function, combination of Eqs.(12) and (13) yields

$$\theta = u\left(\frac{b_A}{q_A}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{b_A}{q_A}\right)^2-1} - u\left(\frac{b_B}{q_B}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{b_B}{q_B}\right)^2-1} \tag{14}$$

Also one can obtain the bending angle θ^* in the case that the extrudate is not swelling.

$$\theta^* = u\left(\frac{a_A}{q_A}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{a_A}{q_A}\right)^2-1} - u\left(\frac{a_B}{q_B}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{a_B}{q_B}\right)^2-1} \tag{15}$$

With the B.C. ($\theta_E=0$ for no swelling), the exit bending angle θ_E becomes

$$\begin{aligned}\theta_E &= \theta - \theta^* \\ &= u\left(\frac{b_A}{q_A}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{b_A}{q_A}\right)^2-1} \\ &\quad - u\left(\frac{b_B}{q_B}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{b_B}{q_B}\right)^2-1} \\ &\quad - u\left(\frac{a_A}{q_A}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{a_A}{q_A}\right)^2-1} \\ &\quad + u\left(\frac{a_B}{q_B}-1\right)\cdot\tan^{-1}\sqrt{\left(\frac{a_B}{q_B}\right)^2-1}\end{aligned} \tag{16}$$

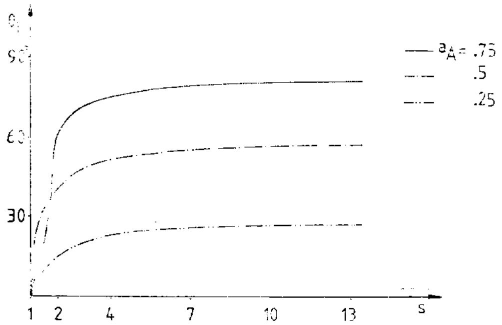

Fig. 4. Exit bendig angle *v.s.* ratio of extrudate swelling.

Since q_A and q_B are functions of a_A and a_B respectively, and also b_A and b_B are functions of a_A, a_B and s, the exit bending angle θ_E depends on the variable s and the phase portion a_A (or a_B). For Newtonian fluid,

$$q_A=(a_1)^3/[(a_1)^3+(a_2)^3] \tag{17a}$$

$$q_B=(a_2)^3/[(a_1)^3+(a_2)^3] \tag{17b}$$

$$\frac{1}{b_A}=1+\frac{a_2}{a_1}\cdot\frac{1}{s} \tag{17c}$$

$$\frac{1}{b_B}=1+\frac{a_1}{a_2}\cdot s \tag{17d}$$

Fig.4 shows a plot of θ_E *v.s.* s. The exit bending angle θ_E increases with the variable s for a given high elastic phase portion a_A. In Fig.5, the exit bending angle θ_E is plotted against the high elastic phase portion a_A for a given variable s. It is found that the exit bending angle θ_E has its maximum value and the maximum value increases with variable s.

Considering that the extrusionability is good when the exit bending angle θ_E is small, it is predicted from Figs.4 and 5 that the variable s should be relatively small and the low elastic phase portion a_B should be larger than the high elastic phase portion a_A.

2.2 Interface Shifting Variable δ

From a macroscopic viewpoint of continuous flow, it is more reasonable to consider that the centerline of flow is continuous (Fig. 6b) than

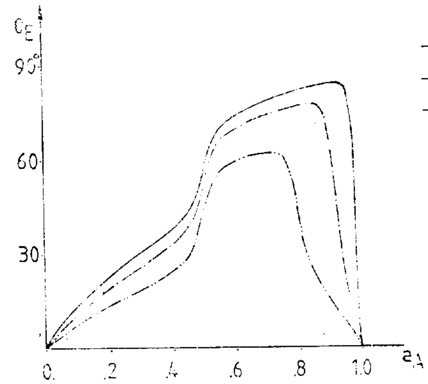

Fig. 5. Exit bending angle *v.s.* high elastic phase portion.

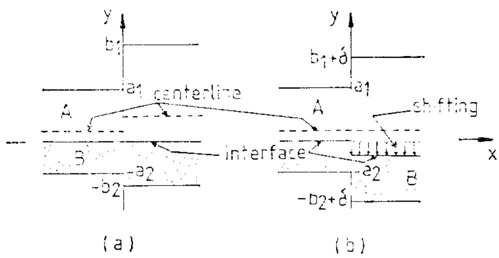

Fig. 6. Geometry of interface shifting parameter (δ).

that the interface of flow is continuous (Fig. 6a) at the die exit. Therefore the interface of flow in die is assumed to move discontinuous to that of swollen flow which has left the die exit. This discontinuity yields the interface shifting variable δ. The value of interface shifting variable δ can be obtained geometrically, as shown in Fig. 6b.

$$\delta=\frac{a_1-a_2}{2}-\frac{b_1-b_2}{2} \tag{18}$$

Also one can easily see that the value of variable δ should be bounded by the widths of two phase A and B. That is,

$$-a_2<\delta<a_1 \tag{19}$$

In Eq. (19), the variable δ can be expressed with the die swell ratio in each phase (b_1/a_1, b_2/a_2). Nevertheless, it cannot be expressed with the variable s into a simple form. At least the absolute value of δ in same phase portion can be predicted to increase with the die swell ratios b_1/a_1 and b_2/a_2. In addition, for δ to be zero, i.e., $a_1-a_2=b_1-b_2$, the values of s are numerous. Values, among them, nearest to zero have rather good extrusionability.

3. Conclusions

As we have seen in theoretical treatment and discussion, if viscous effect is neglected, i.e., only elastic effect is taken into account, the exit bending angle increases with the ratio of extrudate swelling and has its maximum value when the angle is plotted against the phase portion. This maximum value increases with the ratio of extrudate swelling and shifts to the high value of high elastic phase portion as the ratio of extrudate swelling increases.

For a good extrusionability, the die swell ratios in the two phases should be similar and the low elastic phase portion should be greater than the high elastic phase portion. The absolute value of interface shifting, in addition, should be relatively small.

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List of Symbols

a_1, a_2 ; width of phase A and B (cm)

b_1, b_2 ; width of phase A and B after swelling (cm)

a_A, a_B ; portion of phase A and B (dimensionless)

b_A, b_B ; portion of phase A and B after swelling (dimensionless)

q_A, q_B ; fraction of volumetric flow rate of phase A and B (dimensionless)

$(d_j/D_0)_A$, $(d_j/D_0)_B$; die swell ratio in phase A and B (dimensionless)

s ; ratio of extrudate swelling (dimensionless)

m ; slope of interface after die exit (dimensionless)

θ, θ^* ; bending angles (degree)

θ_E ; exit bending angle (degree)

δ ; interface shifting variable (cm)

$u(x)$; unit step function of variable x

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